1 Primary Cosmic Rays

From Astrophysics' point of view, the purposes to research cosmic rays are perhaps:

- 1. What are the *acceleration* sources and physical mechanisms/processes of cosmic rays? (The origin of cosmic rays)
- 2. How do cosmic rays propagate in the universe?

The primary cosmic rays, before impinging on the Earth's atmosphere and subsequently interacting with atomic nuclei in the atmosphere, are composed of from protons to heavier atomic nuclei broadly (roughly, 87% protons, 12% alpha particles (He⁺⁺)). Therefore, *electromagnetic radiations* like X-ray or γ -ray, and neutrinos are NOT included in the cosmic rays in principle.

There are differences between *solar abundance* and *cosmic abundance* concerning the chemical composition consisting of cosmic rays:

- 1. Hydrogen and helium are less compared to those of solar abundance,
- 2. Relatively lighter elements like *Li*, *Be* and *B* are more compared to those of solar abundance,
- 3. Heavier elements are more compared to those of solar abundance.

However, the cosmic abundance (of cosmic rays) is roughly comparable to those found in the Sun.

The energy of cosmic rays ranges also broadly from several tens of MeV to 10^{20} eV (about 50 J, corresponding to the energy of a well-hit tennis ball at 42 m/s).

Table 1: Comparison between the flux of primary cosmic rays (PCRs) [particles/($m^2 \cdot sec \cdot str$)] and the solar abundance ratio (SAR)

Nuclei	Z (atomic number)	flux of PCRs	SAR
р	1	1300	100
He	2	88	8
Li,Be,B	3~5	1.9	$< 10^{-6}$
C,N,O,F	6~9	5.7	0.08
$10 \le Z \le 19$	10~19	1.9	0.02
$20 \leq Z$	$20\sim$	0.53	10^{-3}
Fe	26	0.2	$1.5 \cdot 10^{-3}$

From *Solar System Physics*' point of view, the solar activity and its influence(s) and the Earth's geomagnetic effects have been comprehensively investigated through the cosmic rays as a probe by means of ground-based observations, sounding rockets' and satellites' observations in the upper-atmosphere or the exosphere of the Earth.

Heliospheric Shielding against Cosmic Rays: The solar (magnetic) influence on cosmic rays which impinged from the exterior of the solar system can be estimated by *Parker's model*¹.

¹This model is briefly described in Chapter 4 Planetary Magnetospheres and the Interplanetary Medium, and for the details Introduction to Space Physics (Eds. Kivelson and Russell) is recommended.

The disturbance emerged in the interplanetary magnetic field (IMF) associated with the solar wind is regarded as a scattering medium against the flux of cosmic rays. The scale of interplanetary magnetic disturbance is considered to be 5 nT at the distance around 10^6 km ($\ll 1$ AU=149.60×10⁶ km). Assume that the mean free path of disturbed magnetic field is λ , the speed of cosmic ray particle is v, the speed of the solar wind is V_{sw} , and the number density of cosmic ray at the distance r from the Sun is n(r) then

$$D\frac{\partial n(r)}{\partial r} = V_{sw}n(r). \tag{1}$$

where D is a diffusion coefficient (or diffusivity) (the unit is $[\text{length}^2 \cdot \text{time}^{-1}]$) and is given by $D = 1/3 \cdot \lambda v$. The solution of Eq. 1 is, where $n = n_0$ at r = R (e.g., R is assumed to be close to the heliospheric boundary),

$$n = n_0 \cdot exp[-\frac{V_{sw}}{D}(R-r)].$$
⁽²⁾

The energy (order) of cosmic rays which are scattered "effectively" by disturbed IMF is about 1 GeV. We calculate the diffusivity (D) for the protons ($mc^2=940$ MeV) using the following equations:

$$E^{2} - (mc^{2})^{2} = (pc)^{2},$$

$$pc = \gamma M vc = \frac{1}{\sqrt{1 - (v/c)^{2}}} \cdot \frac{m}{\sqrt{1 - (v/c)^{2}}} \cdot \frac{v}{c} \cdot c = \frac{\beta}{1 - \beta^{2}} \cdot mc^{2},$$
(3)

where $\gamma \equiv c/\sqrt{c^2 - v^2} = 1/\sqrt{1 - \beta^2}$ (Lorentz factor), $\beta \equiv v/c$, E is total energy, m is the rest mass, M is the relativistic mass, p is momentum, and c is the speed of light (~3.0×10⁶ km/s). We are given by v=0.58c for the kinetic velocity of the protons and we also assume that λ =10⁷ km (still ≪1 AU), and thus D=(1/3)·10⁷ [km]·(0.58c)=5.8×10¹¹ km²s⁻¹ is derived.

When R=10 [AU]= 1.5×10^9 km and $V_{sw}=500$ km/s are given, then we get:

$$n \sim n_0 \cdot exp[-\frac{V_{sw}R}{D}] = 0.27 \cdot n_0, \tag{4}$$

where $R \gg r$ is assumed. Notice that $D \sim V_{sw}R$ in the case of the above estimation occurs at and R=10 AU corresponds to approximately the semi-major axis of the orbit of Saturn.

New Depiction about the Heliosphere? : Although Eq. 1 (**Diffusion equation**) is applicable for both charged particles such the cosmic rays and neutral particles in principle, the latest research shows an unexpected picture on around the edge of heliosphere.

Figure 1 shows a data taken by the Energetic Neutral Atom (ENA) detector aboard the Interstellar Boundary Explorer (IBEX) satellite, and a data is considered to reveal an unknown physical process going on in the region close to the *termination shock* of heliosphere. This region is referred as the termination shock because the direction to this region is identified by and with the latest locations of the *Voyager 1 and 2*. Although maps of the 'ribbon' (enhanced by yellow-to-reddish colours) seem to show a luminous body, the ribbon emits no light. Instead, it makes itself known via particles called *energetic neutral atoms* – mainly garden-variety hydrogen atoms. The ribbon emits these particles, which are picked up by IBEX in Earth orbit.

The Solar System is passing through a region of the Milky Way filled with (galactic) cosmic rays and interstellar clouds. The magnetic field of our own sun, inflated by the solar wind into the bubble-like heliosphere, substantially protects us from these things. The size and shape of heliosphere are key factors in determining its shielding power and, thus, how many cosmic rays reach Earth. Contrary to such a conventional picture, the heliosphere in terms of energetic 'neutral' particles seems to be different: Energetic neutral particles (presumably) created at the edge of heliosphere can reach deep into the inner Solar System, i.e. close to Earth and Sun, until being transferred to charged state. This process is called **charge exchange** and expressed as:

$$A + B^+ \to A^+ + B, \tag{5}$$

where B^+ represents (a) component(s) of the solar wind, probably protons (H⁺), and A represents energetic neutral particles, probably a mass-comparable particle to H⁺ so that hydrogen atoms. This charge exchange process occurs in the vicinity of Earth where the IBEX satellite is orbiting.

So far, there is no reasonable explanation to generate energetic neutral particles at the interface between the termination shock and the interstellar space. We will wait further results from the IBEX observations and its data analyses.

Quiz: Survey the detection method of energetic neutral atoms with other detectors, and compare them each other.



1.1 Geomagnetic Effect

Concerning the (charged) particle trajectory along the geomagnetic fields (or in the magnetosphere), the qualification is given as following and shown in Figure 2. (Cf. a quantification has been given by Carl Strørmer, a Norwegian mathematician and physicist, and later Hannes Alfvén, a Swedish plasma physicist, sophisticated the Strørmer's theorem into *guiding centre* approximation.)

1. The trajectory of a charged particle which has a small momentum tends to be bended largely against the geomagnetic fields, thus the particle hardly reaches the Earth,

- 2. A charged particle approaching toward the polar region can easily reach the Earth, almost independent on the magnitude of its momentum,
- 3. A charged particle approaching toward the equatorial region tends to be bended largely, thus a large momentum is needed for the particle to reach the Earth.



Figure 2: Schematic of geomagnetic effect (or shielding) against cosmic rays.

There is apparently the geomagnetic latitude dependence (or is called as magnetic rigidity) concerning the "cut-off" momenta of primary cosmic rays. According to Fig. 4, (a) primary cosmic rays with momentum less than ~ 14 GeV/c (the unit is momentum per charges) cannot impinge anywhere on the Earth, which is exactly the repeat of 1.–3. above, (b) primary cosmic rays approaching to the equator or near-the-equator show being influenced by the *east-west effect* due to eastward rotation of Earth. For the latter, we can explain as following: The particle with charges *ze*, momentum *p*, and velocity v, can be affected by *Lorentz force* in the absence of electric field (or magnetic force) in the magnetic field (and its strength is *B*), which is given as:

$$f = ze(\mathbf{v} \times \mathbf{B}). \tag{6}$$

Consider a charged particle with either westward velocity vector (v) or east ward velocity vector against the Earth rotation which is 'eastward', and approaching to the equator (Figure 3). The magnetic force acting on the charged particle is 'earthward' for westward-moving particle and 'anti-earthward' for eastward-moving particle. The westward-moving charged particle being acted by the earthward magnetic force easily transfer to the guiding centre motion along the geomagnetic field lines. Note that this explanation ignores the gyro-radius (or Larmor radius), $R_g = p/(zeB)$ (for non-relativistic particle, $p=mv_{\perp}$ where v_{\perp} is the velocity component perpendicular to the magnetic field), of the charged particle, so that the charged particles (or chiefly, primary cosmic rays) with vary large momenta can impinge directly on the Earth (i.e., in the atmosphere).

Let us estimate the Larmor radius for primary cosmic rays with very high energy when it is assumed that energy $E \approx pc$ (refer also Eq. 3, and thus the total energy $E \gg mc^2$ in this case).

$$R_g = \frac{E}{10GeV} \cdot \frac{1}{z} \cdot \frac{10^{-6}G}{B} \cdot 3 \times 10^{13} \ [cm],\tag{7}$$



Figure 3: A cartoon for explaining schematically the east-west effect on an equator-approaching primary cosmic ray (or charged particle).

where 1 G(gauss)= 10^{-4} T(tesla). With this equation, we can estimate the *magnetic shielding* effect. For example, very high-energy particles (i.e. cosmic rays) with energy less than (10 GeV/×z) cannot enter inside the atmosphere of the Earth ($R_E \sim 10^9$ cm, $B_E \sim 0.1$ G or 10 μ T).

Quiz: Consider a proton cosmic ray of an energy 14 GeV ($mc^2=0.94$ GeV and thus 14 GeV \gg 0.94 GeV), and assume that the proton approaches just vertically to the geomagnetic fields precisely perpendicular to the equatorial plane.

1. Calculate the Larmor radii of the proton at R=10, 5 and 2.5 R_E (from the centre of Earth). You can use the formula below.

$$B(r,\lambda) = B_0 \left(\frac{R_E}{r}\right)^3 \sqrt{1+3\sin^2\lambda},$$

where $B_0 = 31 \ \mu T.$

2. Discuss the magnetic rigidity of the proton, taking the results obtained above into account.

1.2 Atmosphere-Cosmic Ray Interaction

It has long been known that there is a correlation between the (integral) number flux (generally in [particles/(cm²·sec)] after integrating over energy and solid angle, see also Figure 5) of cosmic rays and atmospheric pressure because cosmic rays impinging on the Earth's atmosphere run through (therefore, 'interact with') the matter of, on the average, 1000 g/cm² (*column density*, defined by the product of length (e.g., cm) and mass density (e.g. g/cm⁻³)), and this column density increases as the atmospheric pressure increases. For example, the number flux of *muons* increases +0.1% as the atmospheric pressure increases +1 mbar (=100 Pa=1 N/m²).

Briefly, muons are so-called *secondary cosmic rays*, which are generated in the Earth's atmosphere via *charged pion decay*:

$$\pi^+ \to \mu^+ + \nu_\mu, \ \pi^- \to \mu^- + \bar{\nu_\mu}.$$
 (8)



Figure 4: Cut-off momenta of primary cosmic rays as a function of geomagnetic latitudes. (a) component of primary cosmic rays normal (vertical) to the ground of the Earth, (b) 45°W and 45°E components of (a). (b) shows a *east-west effect* with which plus(+)-charged particles are said to be predominant in the primary cosmic rays.

and see also Figure 6. The involved process is called as *hadronic interaction* or *strong interaction*².

On the other hand, temperature effect on the number flux of muons is more complex and the formulation is given as

$$\frac{\delta N_{\mu}}{N_{\mu}} = \int_0^{p_0} W_T(p) \delta T(p) dp, \tag{9}$$

where $\delta T(p)$ is the temperature variation at a given pressure p (and its equivalent height), $W_T(p)$ is the temperature effect coefficient at the equivalent height (or the pressure p).

For relatively lower-energy muons (below ~100 GeV), $W_T(p)$ is "minus", i.e. the number flux of muons increases as the temperature decreases. Quantitatively, when the temperature increases under the condition that the pressure is constant, the vertical length of the atmosphere (hence, the column density of the atmosphere) increases. This means that the interaction rate of muons in the atmosphere increases and thus the number flux of muons on the ground decreases as a result. For higher-energy muons (several 100 GeV and more), to the contrary, $W_T(p)$ is "plus". This is due to the relativistic effect, i.e. $t = \gamma t_0$ (where $\gamma = 1/\sqrt{1 - (v/c)^2} = 1/\sqrt{1 - \beta^2}$, *Lorentz factor*, t is time of the observer's frame and t_0 is time of the rest frame) shows that the mean life-time of muon extends. In the seasonal time-scale, the number flux of muons has an anti-correlation with the temperature variation, i.e. the number flux of muons increases as the temperature decreases.

1.2.1 Other Atmospheric Interactions of Cosmic Rays

Later in this compendium, we survey the *electromagnetic interactions* between the cosmic rays and the atmosphere, however, there are other interaction processes which are not described in

²And this issue belongs to Nuclear Physics and Particle Physics, so that it is beyond this course, Solar System Physics, so far.



Figure 5: Differential number flux of cosmic rays as a function of energy. Adapted from *Wikimedia Commons*.

details. We just list: (1) neutrons (production of cosmic ray collisions with atomic nuclei in the atmosphere), (2) isotope production in the atmosphere and subsequent sedimentation (e.g., 14 C), and 3) meteorites and geochemistry (a very long time variation of cosmic rays).

Quiz: Check by yourself that the detection/measuring methods for the above things.

1.2.2 Effects on Electronics

Besides the atmosphere-cosmic ray interaction, cosmic ray's effects on electronics are of large interest for space scientists and space engineers: Cosmic rays have sufficient energy to alter the states of elements in electronic integrated circuits, causing transient errors to occur, such as corrupted data in electronic memory devices, or incorrect performance of CPUs, often referred to as "soft errors" (not to be confused with software errors caused by programming mistakes/bugs). This has been a problem in extremely high-altitude electronics, such as in satellites, but with transistors becoming smaller and smaller, this is becoming an increasing concern in ground-level electronics as well. Studies by IBM in the 1990s suggest that computers typically experience about one cosmic-ray-induced error per 256 megabytes of RAM per month.

To alleviate this problem, the Intel Corporation has proposed a cosmic ray detector that could be integrated into future high-density microprocessors, allowing the processor to repeat the last command following a cosmic-ray event.

Cosmic rays were recently suspected as a possible cause of a Qantas Airlines in-flight incident where an Airbus A330 airliner twice plunged hundreds of feet after an unexplained malfunction in its flight control system. Many passengers and crew members were injured, some seriously. After this incident, the accident investigators determined that the airliner's flight control system had received a data spike that could not be explained, and that all systems were in perfect working order. This has prompted a software upgrade to all A330 & A340 airliners,



Figure 6: Schematics on the fate of primary cosmic rays in the upper atmosphere. Adapted from *Wikimedia Commons*.

worldwide, so that any data spikes in this system are filtered out electronically.

1.3 Temporal Variations of Cosmic Rays

The number flux of cosmic rays observed on the Earth (from the ground up to the upper atmosphere) varies in broad time scales: (1) day-night, (2) seasonal, (3) solar-rotation cycle (\sim 27 days), and (4) solar cycle (11 years). Besides the above, the number flux of cosmic rays is also sensible to 'sporadic' solar activities, e.g. solar flares and coronal mass ejections.

1.3.1 Effects of Solar Activity on Cosmic Rays

27-day cycle It is well known that the number flux of cosmic rays has an 27-day (corresponding to the solar rotation) cycle. This is because the number flux of cosmic rays decreases when the sun-spot region (magnetically active region) on the Sun faces to the Earth (See Fig 7).

11-year cycle With a 11-year cycle, the Sun meets *Solar Maximum/Maxima* and *Solar Minimum/Minima*. The latest solar maximum/minimum were 2001 (maximum) and 1996 (minimum). In general, the number of sunspot increases in association with the (magnetic) activity of the Sun, therefore, the number flux of cosmic rays decreases during the solar maximum.

Forbush Decrease The description is given in Chapter 4. *Planetary Magnetospheres and the Interplanetary Medium*. However, you can find the characteristics of Forbush Decrease in Figure 8.

1.4 History in Cosmic Ray-related Research and the Relationships cross the Other Topics in Solar System Physics

Primary Cosmic Rays: The term "cosmic rays" was coined by Robert Millikan (an American experimental physicist, and Nobel laureate in physics for his measurement of the charge on the electron and for his work on the photoelectric effect) who proved they were extraterrestrial in origin, and not produced by atmospheric electricity as Viktor F. Hess (an Austrian-American



Figure 7: Number of sunspot vs. flux variation of cosmic rays (in %). Observation has been done at Huancayo and the average over $1940 \sim 1941$ is shown.

physicist, and Nobel laureate in physics, who discovered cosmic rays.) had thought. In 1948, observations with *nuclear emulsions* carried by balloons to near the top of the atmosphere by Melvin B. Gottlieb (an American high-energy physicist) and James A. Van Allen (an American space scientist) showed that the primary cosmic particles are mostly protons with some helium nuclei (alpha particles) and a small fraction heavier nuclei.

Air Shower: An *air shower* is an extensive (many kilometres wide) cascade of ionised particles and *electromagnetic radiation* produced in the atmosphere when a primary cosmic ray enters the atmosphere. The term *cascade* means that the incident particle, which could be a proton, a nucleus, an electron, a photon, or (rarely) a positron, strikes a molecule in the air so as to produce many high energy ions (*secondaries*), which in turn create more, and so on.

The original particle arrives with high energy and hence a velocity near the speed of light, so the products of the collisions tend also to move generally in the same direction as the primary, while to some extent spreading sidewise. In addition, the secondary particles produce a widespread flash of light in forward direction due to the *Cherenkov effect*, as well as *fluorescence light* that is emitted isotropically from the excitation of nitrogen molecules. The particle cascade and the light produced in the atmosphere can be detected with surface detector arrays and optical telescopes. Surface detectors typically use *Cherenkov detectors* or *Scintillation counters* to detect the charged secondary particles at ground level. The telescopes used to measure the fluorescence and Cherenkov light use large mirrors to focus the light on PMT clusters.

In 1934, Bruno B. Rossi (a leading Italian-American experimental physicist) reported an observation of near-simultaneous discharges of two *Geiger counters* widely separated in a horizontal plane during a test of equipment he was using in a measurement of the so-called *east-west effect*. In his report on the experiment, Rossi wrote "...it seems that once in a while the recording equipment is struck by very extensive showers of particles, which causes coincidences between the counters, even placed at large distances from one another. Unfortunately, he did not have the time to study this phenomenon more closely." In 1937 Pierre V. Auger (a French physicist), unaware of Rossi's earlier report, detected the same phenomenon and investigated it in



Figure 8: Decrease of cosmic rays' intensity during a geomagnetic storm. The solid line shows the horizontal component of geomagnetic field (H_h) and the dashed line shows the decrease of cosmic rays' intensity (CR in %). These values are averaged over the 25 geomagnetic storms during 1937 to 1945.

some detail. He concluded that extensive *particle showers* (or *air showers*) are generated by high-energy primary cosmic-ray particles that interact with air nuclei high in the atmosphere, initiating a cascade of secondary interactions that ultimately yield a shower of electrons, photons, and muons that reach ground level.

Atmospheric Neutrinos: Three varieties of neutrino are produced when the unstable particles produced in cosmic ray showers decay. Since neutrinos interact only weakly with matter most of them simply pass through the Earth and exit the other side. They very occasionally interact, however, and these atmospheric neutrinos have been detected by several deep underground experiments. The *Super-Kamiokande* in Japan provided the first convincing evidence for *neutrino oscillation* in which one flavour of neutrino changes into another. The evidence was found in a difference in the ratio of electron neutrinos to muon neutrinos depending on the distance they have travelled through the air and earth.

2 Electromagnetic Interactions

In the following section, we survey several physical processes in association with atmosphericcosmic ray interactions. As mentioned previously, *hadronic interaction* will be excluded because of 'beyond Solar System Physics'.

2.1 Electromagnetic Radiation associated with Particle Acceleration – Poynting Vector

The Poynting vector describes the energy flux $[J \cdot m^{-2} \cdot s^{-1}]$ of an electromagnetic field associated with charged particle's motion. It is named after its inventor, an English physicist John H. Poynting. It points in the direction of energy flow and its magnitude is the power per unit area crossing a surface which is normal to it. (The fact that is "points" perhaps contributes to the frequency with which its name is misspelled!) It is derived by considering the conservation of energy and taking into account that the magnetic field can do not work. It is given the symbol S and, in SI units, is given by

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \frac{1}{\mu} \mathbf{E} \times \mathbf{B}.$$
 (10)

where E is the electric field, H and B are the magnetic field and magnetic flux density respectively, and μ is the permeability of the surrounding medium.

For example, the Poynting vector near an ideally conducting wire is parallel to the wire axis, so electric energy is flowing in space outside of the wire. The Poynting vector becomes tilted toward the wire for a resistive wire, indicating the energy flows from the electromagnetic field into the wire, producing resistive *Joule heating* in the wire.

In a classic manner, a Poynting vector has two terms: one is a function of particle's velocity (v) and the other is a function of particle's acceleration rate (\dot{v}) . The second term (a function of \dot{v}) indicates that $\mathbf{E} \perp \mathbf{B}$ and $|\mathbf{E}|=|\mathbf{B}|$, thus a radiated electromagnetic wave. The angle between the direction of particle's motion and the direction of electromagnetic radiation is θ , the azimuthal angle around the particle's motion is ϕ , then the energy per unit time, P [J·s⁻¹], radiated in association with an electromagnetic wave is, when $v \ll c$,

$$P = \frac{e^2}{c^4} \int_0^{2\pi} d\phi \int_{-1}^1 d(\cos\theta) \dot{v}^2 \sin^2\theta = \frac{4\pi}{3} \cdot \frac{e^2}{c^4} \cdot \dot{v}^2,$$
 (11)

where $\sin^2 \theta$ means that a bi-polar radiation is efficient. From $F = ma = m\dot{v}$ and for the same strength of F(|F|), $\dot{v} = |F|/m$. This means that a radiated energy per unit time, P, is inversely proportional to m^2 , thus P associated with electron's motion is 3.4×10^6 times larger than that of proton's motion due to $m_p/m_e \sim 1800$.

When v is close to c or $v \approx c$, the rest mass should be replaced by relativistic mass, γm , where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Hence

$$P = \frac{4\pi}{3} \cdot \frac{e^2}{c^4} \gamma^4 \cdot (|a_{\perp}|^2 + \gamma^2 |a_{\parallel}|^2),$$
(12)

where a_{\perp} and a_{\parallel} are the components perpendicular or parallel to v respectively.

2.2 Bremsstrahlung

This is a "braking radiation" in German, and electromagnetic radiation produced by the acceleration of a charged particle, such as an electron, when deflected by another charged particle, such as an atomic nucleus. The term is also used to refer to the process of producing the radiation. Bremsstrahlung has a continuous spectrum. This electromagnetic radiation may also referred to as *free-free* radiation. This refers to the radiation that arises as a result of a charged particle that is free both before and after the deflection (acceleration) that causes the emission. Any radiation due to the acceleration of a charged particle, which includes *synchrotron radiation*, may be referred to as this radiation, however, it is frequently used in the more narrow sense of radiation from electrons stopping in matter.

In some manner, the term *outer bremsstrahlung* is used in cases where the energy loss by radiation greatly exceeds that by *ionisation* as a stopping mechanism in matter. This is seen clearly for electrons with energies above 50 keV.

If a particle of charge q experiences an acceleration **a** which is collinear with its velocity **v**, the angular distribution of the bremsstrahlung is

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \cdot \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}.$$
(13)

where $\beta = v/c$. Hence the total power emitted, i.e. integrated by $\int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) \sin^2 \theta$ is

$$P = \frac{\mu_0 q^2 a^2 \gamma^6}{6\pi c},\tag{14}$$

where γ is the Lorentz factor.

On the Earth, in particular in the auroral region where energetic (above 1 keV) electrons precipitate and thus cause auroral phenomena, bremsstrahlung is produced as a X-ray emission. At upper atmosphere where auroral phenomena occur, so-called *ionosphere*, when a primary, energetic particles (e.g. electrons) interact with nuclei in the atmosphere, electrons in atoms are kicked out and simultaneously the atoms are excited (i.e. *impact ionisation*), and subsequently released/kicked-out electrons lose their energies by radiating photons (in form of X-ray, for example) when their paths are deflected by Coulomb collisions.

The ionosphere is a region where atmospheric ionisation occurs, in particular predominantly by solar ultraviolet (UV) photons and this process is known as *patronisation*. However, precipitating energetic electrons are occasionally more important for the atmospheric ionisation process.

The density gradient of atmospheric electrons results in a simple relationship between an incident electrons' energies and the altitude at which the electrons interact with nuclei in the atmosphere and hence cause ionisation. Furthermore, the amount of energy loss due to bremsstrahlung is simply related to the *ionisation loss*.

Cyclotron and Synchrotron Radiations: In the presence of magnetic field, electrons show a gyrating motion (or gyration) caused by Lorentz force (F=ev/B). The gyroradius, $r_g=p/eB$, is proportional to the momentum (p) of electron. When $v \ll c$, p=mv, and thus the gyroperiod $T=2\pi r_g/v$ is constant: $T_c=2\pi/\omega_c=2\pi \cdot (m_e/eB)$, where $\nu_c=eB/2\pi m_e$ and $\omega_c=eB/m_e$ are cyclotron frequency and cyclotron angular frequency respectively. From the charged particle's gyration, a radiation with a constant frequency is emitted and is called cyclotron radiation. The electric field vector (\mathbf{E}) of radiation is parallel to the plane of gyration orbit, and the most intensive radiation is observed normal to the plane of gyration orbit.

When v is close to c and the energy of electron (E_e) becomes larger, the relativistic treatment will be needed: instead of p=mv, $pc=(\gamma m_e) \cdot (\beta c) \cdot c=\gamma\beta m_e c^2$ should be used. As a result, the gyro-period, T, is no longer a constant and should be modified as

$$T = \frac{2\pi r_g}{\beta c} = 2\pi \cdot \frac{\gamma \beta m_e c^2}{eB} \cdot \frac{1}{\beta c} = \frac{2\pi \gamma m_e c}{eB}.$$
 (15)

and the gyro-period of relativistic case becomes $T_c \times (\gamma c)$. This relativistic radiation is called *synchrotron radiation*, and the relationship between the relativistic acceleration and the radiation intensity (or energy loss per unit time of electron) is given as (cf. Eq. 12)

$$-\frac{dE_e}{dt} = \frac{4\pi}{3} \cdot \frac{e^2}{c^4} \gamma^4 \cdot (|a_\perp|^2 + \gamma^2 |a_\parallel|^2).$$
(16)

Furthermore, inserting $a_{\perp} = F/(\gamma m_e)$ where F is the Lorentz force in the presence of magnetic field B exerting on the electron $(F=e \cdot (\beta c) \cdot B \sin \theta_B)$, where θ_B is the angle between the direction of magnetic field and the direction of electron motion). Hence

$$-\frac{dE_e}{dt} = \frac{4\pi}{3} \cdot \frac{e^2}{c^4} \gamma^4 \cdot \frac{e^2 \beta^2 c^2 B^2 \sin^2 \theta_B}{\gamma^2 m_e^2} = \frac{c^2}{2} \cdot \sigma_T \cdot \gamma^2 \beta^2 B^2 \sin^2 \theta_B,$$
 (17)

where σ_T is the *Thomson (total) cross section* (= $8\pi/3 \cdot r_0^2$, and r_0 , the classical orbital radius of electron or the *Thomson differential cross section*, is $e^2/(m_ec^2)$). To calculate $-dE_e/dt$, we should take an average over the distribution of θ_B ($f(\theta_B)d\theta_B$).

It is known that periodic pulsations from Jupiter, presumed to be associated with the synchrotron radiation by relativistic electrons, have been observed. These relativistic electrons are considered to being rotating near the Jovian equatorial plane. This rotational motion of electrons might be caused by the fast rotation of Jupiter and an extremely strong Jovian magnetic field feeds electrons to be relativistic.

2.3 Cherenkov Radiation

This topic is dealt in Chapter 7. Solar and Atmospheric Neutrinos.

2.4 Cherenkov and Fluorescent lights in the Atmosphere

These topics are dealt in Chapter 7. Solar and Atmospheric Neutrinos.